## 2021 James S. Rickards Fall Invitational

Name:	
School:	
ID∙	

- 1. \_\_\_\_\_ Nitish's pet stone, Rico, is in the shape of a cube. If the side length of Rico is 3 centimeters, how long is Rico's space diagonal, in centimeters?
- 2. \_\_\_\_\_\_ If the side length of Rico is 30 centimeters, what is the volume of Rico in cubic meters?
- 3. \_\_\_\_\_ Prabhas the probability master rolls 3 fair dice. What is the expected value of the sum of the three numbers facing up?
- 4. \_\_\_\_\_ Next Prabhas rolls 2 fair dice. What is the expected value of the product of the two numbers facing up?
- 5. \_\_\_\_\_ How many zeroes are in the binary representation of 4095?
- 6. \_\_\_\_\_ What is the instantaneous slope (derivative) of the function f(x) = (x+1)(x+2) at x = 6?
- 7. \_\_\_\_\_ Evaluate the following expression.

$$\frac{\sum_{i=1}^{20} i^3}{(\sum_{i=1}^{20} i)^3}$$

- 8. \_\_\_\_\_ Dylan randomly chooses a natural number n greater than 1 but less than 22. What is the probability that a rose curve can be formed with n petals?
- 9. \_\_\_\_\_ Compute 101 · 99.
- 10. \_\_\_\_\_ Find the sum of the eigenvalues of the following matrix.

$$\begin{bmatrix} 4 & 5 & 8 & 20 \\ 2 & 9 & 5 & 9 \\ 2 & 4 & 6 & 8 \\ 34 & 21 & 9 & 1 \end{bmatrix}$$

- 11. \_\_\_\_\_ Find  $3^7$ .
- 12. \_\_\_\_\_ How many ways can 6 differently colored beads be placed on a bracelet, given that rotations and flips of a specific bracelet are considered congruent?
- 13. \_\_\_\_\_ Evaluate the following product:  $94 \cdot 96$ .
- 14. \_\_\_\_\_ Find the sum of the squares of the first ten whole numbers.
- 15. \_\_\_\_\_ Find the largest prime that 2021! is divisible by.
- 16. \_\_\_\_\_ Evaluate the following sum.

$$\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \ldots + \frac{1}{575}$$

- 17. \_\_\_\_\_ How many diagonals does a convex dodecagon have?
- 18. \_\_\_\_\_ Evaluate  $\cos(\frac{\pi}{2})$ .
- 19. \_\_\_\_\_ If January 1<sup>st</sup> of 2020 was a Wednesday, what day of the week will January 1<sup>st</sup> of 2022 be?
- 20. \_\_\_\_\_ Find the product of the eigenvalues of the following matrix.

28	62	30	2
17	8	4	5
14	31	15	1
9	6	5	10

- 21. \_\_\_\_\_ How many different types of regular polygons have integral angle measures?
- 22. \_\_\_\_\_ If Tanmay has 6 pairs of shorts, 6 socks, and 10 shirts, how many different outfits can he wear that consist of a pair of shorts, 2 socks, and one shirt?
- 23. \_\_\_\_\_ A certain sequence S exists such that the sum of the first n elements of S is equivalent to  $n^3$ . Find the 10<sup>th</sup> element of S.
- 24. \_\_\_\_\_ Evaluate the following sum:

 $\sum_{28}^{73} 2n - 1$ 

- 25. \_\_\_\_\_ How many factors does 2021 have, excluding itself?
- 26. \_\_\_\_\_ On any given day, Tanmay either sleeps 4 or 12 hours. If over the course of a 30 day month, he sleeps 208 hours, how many days did he sleep 12 hours?
- 27. \_\_\_\_\_ Let A be the LCM of 45 and 55. Let B be the GCF of 45 and 55. Find the product AB.
- 28. \_\_\_\_\_ Find the units digit of  $482^{29} + 3986^{10} + 325^{257}$ .
- 29. \_\_\_\_\_ Find the surface area of a rectangular prism with side lengths as the roots of the following polynomial:

$$10x^3 - 307x^2 + 496x - 87 = 0$$

- 30. \_\_\_\_\_ What is the minimum value of f(x) for the following function:  $f(x) = x^2 4x + 20$ ?
- 31. \_\_\_\_\_ Karthik will only help his 3 friends (Tanmay, Nitish, and Akhil) out with 5 problems on the bus ride to a math competition. If 2 questions to Tanmay, 3 questions to Nitish, and none to Akhil is one possible way in which he may help his friends, in how many different ways can he offer his services?
- 32. Let f(word) be equivalent to the amount of different ways one can arrange the letters in the word. For example, f(ERIC) is equal to 24. Evaluate the following expression:

$$\frac{f(SHAWN) + f(GUS)}{f(TROY) + f(ABED)}$$

- 33. \_\_\_\_\_\_ Shrung needs to pack his bags for Indiana. Each of Shrung's shirts is a 3 inch by 3 inch by 1 inch rectangular prism, and the inside of his suitcase is a 4 inch by 9 inch by 5 inch rectangular prism. He will fit as many shirts as possible in his suitcase, and then use the rest of the space for his 1 inch by 1 inch by 1 inch USB drives filled with Zoom backgrounds. How many USB drives will he bring along on his journey?
- 34. \_\_\_\_\_ Neal Caffrey has a 2 mile radius in which he can freely roam in Manhattan. Find the amount of square miles within which he can roam, rounded to the nearest integer.
- 35. \_\_\_\_\_ Shubham swims 2 miles on every day of the week other than Sunday. Given that 2021 ends on a Friday, how many miles did Shubham swim in 2021?
- 36. \_\_\_\_\_ Ananya has a morbid fear of birds, as she finds them creepy. Upon finding a bird in her cubic room (with a 10 foot side length), she tethers the bird to a 10 foot long leash in the bottom right corner of the room. How many cubic feet of the room are safe for Ananya?
- 37. \_\_\_\_\_ The prime factorization of 2016 can be expressed as  $a^p \cdot b^q \cdot c^r$ . Find ap + bq + cr.
- 38. \_\_\_\_\_ Find the area of the shape created by the following equation: |x| + |y| = 10.
- 39. \_\_\_\_\_ Evaluate 20 + 21.
- 40. \_\_\_\_\_ Mozzie randomly shuffles a deck of cards. What is the probability that the Queen of Hearts is the fifth card from the top?